Here, we present a brief review of the current status of the Maunder minimum study. The Maunder minimum is considered as an example of occasionally occurring Grand minima, when the solar dynamo was in a special mode. We review available sets of direct and indirect data covering the period during and around the Maunder minimum. The start of the minimum was very abrupt and was followed by a gradual recovery of the activity. The data suggest that while the sunspot activity was greatly suppressed during the deep phase of the minimum, the cyclic dynamo kept working around the sunspot formation threshold level, leading to seemingly sporadic occurrence of sunspots. The majority of proxy data depict the dominant 22-year periodicity during the Maunder minimum with the sub-dominant 11-year cycle. The length of the cycles was probably slightly enhanced. We also discuss theoretical models and speculations concerning the solar dynamo as well as the heliosphere during the Maunder minimum. Comparison with other minima (Spörer and Dalton) suggests that these features are common.

1. Introduction

Solar magnetic activity usually exhibits cyclic behavior with about 11-year period, which is also subject to great secular variations. While the contemporary level of the activity is high, the normal cyclic activity is sometimes interrupted by periods of unusually low activity known as Grand minima. The most recent Grand minimum is the so-called Maunder minimum (MM) that took place between 1645 and 1715 (the deep phase in 1645–1700). The idea of “a prolonged sunspot minimum” was suggested already in the 19th century. The initial concept of a Grand minima was introduced based on rather nonsystematic and indirect evidence and observations (see a brilliant review of the history of the Maunder minimum discovery and underlying physical and astronomical ideas). However, it took almost a century before this idea became widely accepted after careful consideration of
other related solar proxy data, including cosmogenic $^{14}$C. Although the very existence of the MM is beyond doubt now, it is much less clear what exactly had happened during the MM and whether the MM is similar to other Grand minima. The existence of such long (several decades) periods of suppressed solar activity is very important for understanding dynamo processes in the Sun and other stars. A particular important question is whether the physical mechanism responsible for the solar activity cycle was still operating during the MM, and if yes — in what mode? Other Grand minima of solar activity are known (e.g., those called after Spörer, Wolf, etc.) using cosmogenic isotope data, but the MM is very special since it is the most recent one and amazingly well covered by direct and indirect data. Since the 1980s, from archival data on instrumental solar observations performed by the French astronomical school in (17–18)th centuries the solar activity have been restored. Together with precisely measured cosmogenic isotope data this allows for a systematic quantitative study of solar activity during the MM.

Here we aim to review the recent observational facts and theoretical speculations related to the MM. In the present paper, we first consider various observational data and evidence (Sec. 2) and present theoretical models to deal with the solar dynamo and heliosphere during a Grand minimum (Sec. 3). Then, we compare the available data for other known Grand minima with the pattern of the MM (Sec. 4). Conclusions are summarized in Sec. 5.

2. Observational Data

Here, we consider the bulk of direct and indirect data available for the MM. These data form proxies for different solar and heliospheric parameters. In particular, sunspot number is a proxy for the toroidal magnetic field in the convection zone, aurorae provide information about transient interplanetary phenomena and local heliospheric state, and data on cosmogenic isotopes $^{14}$C and $^{10}$Be are governed by the global heliospheric modulation of cosmic rays.

2.1. Solar observations

By initial definition, the MM is a peculiar epoch in solar activity that took place at the time of Louis XIV, during which sunspots were almost absent. After a systematic investigation of archive data by Ribes and Nesme-Ribes
we know that, during the decade 1660–1670, the French astronomer Picard supported by the interest of Louis XIV observed the Sun as often as it was possible at that time and fixed the results in a way reasonably comparable with modern standards of scientific presentation. The result is that only one sunspot was observed during that period. The corresponding conclusion is twofold. On one hand, the level of solar activity was extremely weak in comparison with periods after and just before the MM. On the other hand, the high reliability of the data implies that the only observed sunspot manifests some level of solar activity during the most deep phase of the MM. The point is that the sunspot formation is associated with a substantial toroidal magnetic field located somewhere deep inside the Sun. A maintenance of such field needs some excitation mechanism which is usually identified with solar dynamo which in turn is usually cyclic.

At present, we know that sunspots were observed during 368 days within the deep MM (1645–1700), which is less than 2% of all days during the MM (see Fig. 1). Note that about 95% of days during the deep MM were covered by reported solar observations. A careful analysis and conservative consideration of the available data show that, despite uncertainties in the data, the level of sunspot activity was indeed extremely low during the MM: yearly sunspot numbers are below 4 for the deep MM, and below 8 for the cycle 1700–1712. While the sunspots were very scarce in the first half of the MM, the recovery of solar activity after 1670 became more and more noticeable in sunspot data and a clearly distinguishable regular solar cycle took place in 1698–1712. Because of the scarcity of sunspot occurrence, standard time series analysis methods cannot be applied to study the question whether the sunspot activity was still cyclic during the MM. By means of a special method for analysis of the occurrence of sparse events, Usoskin et al. studied time clustering of sunspot occurrence (see Fig. 1). This figure shows clustering of sunspot occurrence as a function of the scale (see Ref. 14 for details). One can see that there are two major clusters, around 1660 and 1680, which are persistent through all scales. Together with the neighboring sunspot cycle maxima in 1640 and 1705, it implies that the dominant 22-year periodicity in sunspot activity was present during the MM. Some sub-clustering is visible at small scales.

It is important that sunspot formation is a threshold effect and the absence of spots during the major part of the MM does not necessarily imply switch-off of the dynamo process. The solar cycle probably existed but at a sub-threshold level and was only barely visible in sunspot data.
Another interesting fact is that it is possible to build butterfly diagrams for some periods of the MM, thanks to the archives of the French astronomy school. The butterfly diagram was strongly asymmetric during the MM with the majority of spots observed in the Southern solar hemisphere. This fact is very important for understanding the Grand minimum scenario as discussed in Sec. 3.1.

Basing on archive observations of Mutton, Richerd, Picard and La Hire, Ribes et al. presented a time series for the apparent solar diameter for the MM period. A wavelet analysis of these data shows that an approximately 11-year cyclicity exists in the apparent solar diameter data. Note however, that the apparent solar diameter observations are much more uncertain than sunspot data, and the level of cyclic variations is very low. Accordingly, the question of the cyclic variations of the solar diameter during the MM is still controversial in general.

The data available suggest that solar magnetic cyclic activity might have been in a special mode during the MM, which is different from the modern activity.

2.2. Cosmogenic isotopes

Cosmogenic isotopes provide the most extendable indirect data on the cosmic ray flux, the state of the heliosphere, and hence on the solar magnetic
activity during the past. The most commonly used cosmogenic isotopes are radiocarbon (i.e., $^{14}$C) and $^{10}$Be, which are measured in tree-rings and in ice cores, respectively. Both tree-rings and ice cores form stratified structures and retain the time variations of the abundance of isotopes in each layer.

Cosmogenic isotopes are produced in the Earth’s atmosphere mainly by galactic cosmic rays, which originate from outside of the heliosphere and are modulated by the solar wind and interplanetary magnetic field. The basics of the modulation process are well understood (see, e.g., Refs. 20 and 21), and the attenuation level of cosmic rays in the heliosphere depends on the strength and level of turbulence of solar magnetic field and on the global structure of the heliosphere. Basically, the flux of cosmic rays impinging on the Earth is inversely correlated with the solar activity, but shows also variations depending on, for example, the polarity of solar magnetic field. These variations may become important during the Grand minimum. Generally, cosmogenic isotopes can serve as an index of the poloidal magnetic field which is carried out by the solar wind and fills the heliosphere. This is fully applicable for the 11-year cycle averaged data, while inside the solar cycle, an important role is played also by the tilted heliospheric current sheet and drift processes.

$^{14}$C and $^{10}$Be are produced in the atmosphere as a result of nuclear reactions of cosmic rays with the atmospheric nuclei. Then $^{14}$C is oxidized to form carbon dioxide and circulates within the carbon cycle between different reservoirs, some of which are very inertial, and it gets eventually absorbed by trees by means of photosynthesis. On the other hand, $^{10}$Be becomes attached to aerosols, precipitates with snowfall and is accumulated in the ice in polar regions. These differences in the transportation system decide the advantages and disadvantages of each isotope, and sometimes cause different behaviors in the time series of the two isotopes. In this way, using both $^{14}$C and $^{10}$Be, we can trace the history of cosmic rays, the heliosphere and the Sun. Since cosmogenic isotopes are measured contemporarily, their data series are homogeneous, i.e., with nearly constant quality and resolution throughout the recorded period, contrary to direct observables (sunspots, aurorae) which were quite unevenly observed/recorded in early times. Due to distinguishable layers in tree-rings and ice cores, high-temporal resolution can be achieved (one year in tree rings and 2–3-year in ice cores).

Here we review two $^{14}$C records and one $^{10}$Be record from around the time of the MM. It was first shown by Stuiver23 that variations of the $^{14}$C content are inversely correlated with the level of solar activity. The first systematically measured annual $^{14}$C data for the epoch since 1510 were
reported by his group in 1993\textsuperscript{24} and greatly improved in 1998.\textsuperscript{25} A record of $^{10}$Be in polar ice as a tool for investigating the solar activity was made by Beer et al.\textsuperscript{26,27}

The $^{10}$Be record obtained from the Dye-3 site\textsuperscript{27} provides annual data since 1424 AD and shows significant 11-year cyclic variation, persisting through the MM.\textsuperscript{28} On the other hand, $^{14}$C data exhibit quite a different cyclic behavior during the MM. Peristykh and Damon\textsuperscript{29} reported the disappearance of the 11-year cyclic variation during the MM. They divided the $^{14}$C record\textsuperscript{24} into three periods with 90-year interval: before (1540–1630), during (1630–1720), and after (1715–1805) the MM, and analyzed them using the Maximum Entropy method. A significant approximately 11-year signal was found before and after the MM, but only signals of 24.2, 15.6, and 6.22 years were detected during the MM with the lack of the 11-year cycle.

Recently, a new independent record of precisely measured biennial $^{14}$C data from a Japanese cedar tree was obtained for the MM by Miyahara et al.\textsuperscript{30–32} Using these two independent $^{14}$C records (Fig. 2), a cross spectrum of the frequency analysis was obtained (Fig. 3) aiming to remove regional climate effect and measurement systematic errors of $^{14}$C data.\textsuperscript{30} In this cross spectrum, two main periods of 13–15 and 24–29 years were detected, and it was suggested that the 11-year cycle and consequentially the 22-year cycle were lengthened by a few years.

Figure 4 shows wavelet spectra of the two $^{14}$C records from 1617 to 1745 AD. The spectra were obtained using the S-transform.\textsuperscript{33} The spectra consistently show the period of about 14 years from 1660 to 1715 AD. The quasi-periodic signal of about 26 years, which probably corresponds to the lengthened 22-year cycle, is seen through the period in both spectra. However, the 11-year signal is quite weak from 1640 to 1660 AD, suggesting strong suppression or discontinuity of solar cyclic activity. A lengthening of the 11-year cycle is also visible in $^{14}$C data around the Dalton minimum (Fig. 5), while other data sets (sunspot, aurora,\textsuperscript{22} and $^{10}$Be\textsuperscript{24}) yield controversial results for that period.

2.3. Magnetospheric phenomena

Magnetospheric disturbances can be observed by means of aurorae (polar lights), which are caused by transient heliospheric phenomena such as coronal mass ejections or high-speed solar wind streams. Therefore, easily observable aurorae, records of which can be found in archives, form a proxy
Fig. 2. Variation of $^{14}$C content in tree-rings around the time of the MM. The solid line shows the biennial record from Refs. 30 and 32 and the dotted line shows the annual record from Ref. 25.

Fig. 3. Cross spectrum of the S-transform of the $^{14}$C records shown in Fig. 4. Horizontal lines correspond to the periods 11, 13, and 15 years, respectively. The solid line denotes the boundary effects.
Fig. 4. Spectra of the S-transform of the $^{14}$C records for the MM. Left and right panels correspond to the $^{14}$C series by Miyahara et al.\textsuperscript{30–32} and by Stuiver et al.\textsuperscript{25} Horizontal lines correspond to the periods 11, 13, and 15 years, respectively. The solid lines denote boundary effects.

Fig. 5. Spectrum of the S-transform of the $\Delta^{14}$C record\textsuperscript{25} for the period from 1650 to 1850 AD. Horizontal lines correspond to the periods 11, 13, and 15 years, respectively. The solid lines denotes the boundary effects.

for major local interplanetary disturbances and, hence, for the solar magnetic activity over both the solar cycle and longer time intervals (see, e.g., Refs. 15 and 35). While the 11-year cycle is dominant in the auroral series during normal solar activity times, it was suppressed during the MM.\textsuperscript{35}
Fig. 6. Auroral activity at mid-latitudes according to Refs. 36 (solid “KP88” line) and 37 (dotted “S92” line). All data are five-year smoothed.

Instead, significant peaks with longer periods of approximately 19–20, 25, and 15 years appear in the power spectrum during and around the MM.35

Figure 6 presents two five-year smoothed series of auroral activity observed in central Europe: combined series36 from sites with latitude below 55°N (mostly Czech and German sites) and solely German observations.37 Aurorae were very rare (0–4 aurorae/year) during the MM, in accordance with the suppressed solar activity.4,5 However, it is important to note that some aurorae were still observed at mid-latitudes, implying that the solar activity did not vanish completely even during the deep phase of the MM. Periods of increased auroral activity agree fairly well with sunspot occurrence during the MM (see Sec. 2.1). This implies a dominant 22-year cycle in auroral activity during the MM in phase with the similar pattern in sunspot activity. A small increase in the auroral series occurred in 1695, corresponding to the isolated sunspot group and denoting a subdominant 11-year cyclicity at the end of the MM. Accordingly, the time behavior of auroral occurrence in central Europe, and, hence, of the major transient irregularities in the inner heliosphere, is in good agreement with sunspot activity during the MM.

2.4. A grand minima scenario

Let us summarize the main features of the MM according to the available data. The MM is considered as an example of Grand minima when the intensity of solar activity cycle diminishes drastically and a specific state of solar activity occurs. A Grand minimum epoch is substantially longer than a solar activity cycle, and therefore cannot be related to as an
unusual cycle. Transition from normal activity to the MM was very abrupt compared to the typical time scale of the solar cycle. On the other hand, the recovery of activity at the end of the minimum was gradual and took several decades. Because of the gradual recovery, the total duration of a minimum is not well-defined (Eddy defined the MM duration as 1645–1715). Roughly we can define the deep phase as 1645–1700, when sunspots occurred seemingly sporadically without an apparent cyclic behavior, while the whole minimum was extended until ca. 1712, including the very tiny but regular solar cycle 1700–1712. Cyclic activity did not completely disappear even during the deep minimum but was reduced to a level that is sub-threshold for sunspot formation. The solar cycle is clearly visible at the end of the MM, see cycle in 1700–1712. Because of the scarcity of the sunspot occurrence, traditional methods of spectral analysis (e.g., wavelet analysis) applied to a particular data set (e.g., sunspot data) can be unable to reveal the periodicities in the deep phase of the MM. On the other hand, special methods of data processing demonstrate that a cyclic behavior was present during the whole MM. Both nominal 11- and 22-year cycles can be found inside Grand minima with a possible lengthening of the periods. A substantial North–South asymmetry appears typical for the Grand minima events. During the MM, sunspots appeared predominantly in the Southern hemisphere with the apparent lack of spots in the Northern hemisphere. The 22-year cycle was notable in the sunspot data, implying that the magnetic Hale cycle kept working during the MM, with the phase locked.

It is important that, despite the seemingly sporadic occurrence of sunspots, the regular dynamo kept working although in a special mode.

Heliospheric parameters (solar wind, interplanetary magnetic field, and the size of the heliosphere) were reduced during the MM. Suppressed auroral activity implies that heliospheric transient phenomena were quite rare. This conclusion is consistent with results available for the Spöerer minimum.

It looks plausible to suggest that, apart from typical Grand minima, weaker and shorter suppressions of the solar activity sometimes take place. These might include, for example, the Dalton minimum and a minor event that occurred at the end of 19th century.

3. Theoretical Speculations

Here we would like to note that the interpretation of the above data can hardly lead to a unique result. We try to avoid pushing forward a specific interpretation favoring our own scientific preferences but rather to present a survey of the relevant theoretical ideas and speculations.
3.1. Solar dynamo

Let us briefly remind the basic scheme of solar dynamo. The solar activity cycle is presented as a manifestation of a dynamo wave propagating inside the Sun. In first approximation, the dynamo wave corresponds to the concentration of the toroidal magnetic field propagating from middle latitudes towards the solar equator. As was noted by Larmor as early as in 1919, the only realistic way to excite such magnetic field in the framework of Maxwell equations is associated with the Faraday induction effect. However, according to the Lenz rule, toroidal magnetic field $B_T$ cannot be excited without poloidal magnetic field $B_P$.

A particular scheme of solar dynamo suggests a physical mechanism connecting toroidal and poloidal magnetic fields. An obvious way to obtain toroidal magnetic field from poloidal one is the solar differential rotation. It is, however, much more difficult to obtain $B_P$ from $B_T$. Parker\textsuperscript{39} suggested that this can be done by means of cyclonic motions in the solar convective zone. A joint action of Coriolis force and density gradients results in an excess of right-hand vortices in one hemisphere and left-hand vortices in the other hemisphere. In turn, a component of the mean magnetic field $B$ parallel to the mean electric current $J$ appears due to this excess. A consistent theory of this effect was developed in 1960s by Krause and Rädler\textsuperscript{40} who used the notation $\alpha$ for the proportionality coefficient between $B$ and $J$. This effect is known now as the $\alpha$-effect. This scheme results in self-excitation of a dynamo wave similar to that one known from observations.

The toroidal magnetic fields in Northern and Southern solar hemispheres usually have opposite polarities. This toroidal magnetic field configuration is referred to by theoreticians as dipolar. The Maxwell equations admit however another configuration with the toroidal magnetic field of the same polarity in both hemispheres, which is called quadrupolar configuration. In practice, phases of the dynamo waves propagating through Northern and Southern hemisphere can be shifted in respect to each other. This displacement can be presented as an admixture of the quadrupole configuration with the dipole.\textsuperscript{41}

The toroidal magnetic field is hidden inside the solar convective zone and is inaccessible for direct observation. Fortunately, the toroidal magnetic field can be traced by sunspots. On one hand, the sunspots are not an inevitable component of solar dynamo. One can imagine a star with a dynamo wave propagating somewhere inside the convective zone where due to some reason the sunspot production is strongly suppressed. It would
be very difficult to recognize the existence of toroidal magnetic field on such a star. In contrast, poloidal magnetic field is present on the solar surface directly. On the other hand, the toroidal magnetic field inside the Sun known via sunspot data is much more intense than the relatively weak poloidal magnetic field. The most spectacular data concerning solar and stellar activity cycles are indirect and represent the toroidal magnetic field behavior. Direct data related to the poloidal magnetic field behavior are more obscure. As a matter of fact, comparisons between dynamo models and observations are based mainly on sunspot data. Cosmogenic isotope data are particularly important because they reflect properties of the poloidal magnetic field, i.e., they are complementary to the sunspot data.

The importance of Grand minima events for the solar dynamo theory is twofold. On one hand, the very existence of Grand minima is a challenge for the theory. For instance, Grand minima may be simulated in some numerical models (see, e.g., Refs. 42–45) of the solar dynamo but the theoreticians yet cannot straightforwardly explain why the Grand minima occur. It is very important to select those features of the Grand minima phenomenology which are relevant for a confrontation with solar dynamo models. On the other hand, studying the Grand minima allows understanding of how the solar dynamo machine works in unusual regimes.

In principle, one could suppose that the occurrence of a Grand minimum can be related to a suppression of sunspot formation without changing the dynamo mechanism itself. This possibility is unfavorable for dynamo interpretation and can be declined because of the fact that the magnetic activity recovery was strongly asymmetric at the end of the MM\(^46\) (see Sec. 2.1). This argument is however not completely decisive because of the threshold nature of sunspot formation, which could amplify a small random North–South asymmetry of the toroidal magnetic field to a seemingly asymmetric butterfly diagrams. The pattern followed from cosmogenic isotope data and auroral records during the MM (Sec. 2.2) rejects this interpretation on a more solid way. This indicates that not only sunspot formation but also the global solar/interplanetary magnetic field was reduced during the MM.

As a result, we conclude that Grand minima are associated with some disturbances in the solar dynamo machine, although this machine keeps working during a Grand minimum. The cyclic component of the activity during the MM is most pronounced in the cosmogenic isotope data while sunspot and aurora data provide only indicative support. Let us summarize, according to the spectral analysis of isotope data, the main features which are crucial for the dynamo interpretation. The nominal 11-year signal is
intermittent around the Maunder minimum in the $^{14}$C records, sometimes it is not visible or is discontinuous. A signal for the nominal 22-year cycle is weak but regular in $^{14}$C data. This may imply that during the MM something happened with the structure of poloidal field oscillations. Simultaneously, the configuration of toroidal magnetic field was strongly asymmetric with respect to the solar equator. The scenario of the MM (generally valid also for the Dalton minimum) suggests that the dynamo is most suppressed in the beginning of the minimum, followed by a gradual recovery. Note that such a behavior is consistent with a stochastically forced return map model of the dynamo.47

We note that some events in the solar activity history may be interpreted as not fully successful attempts of the solar dynamo machine to switch into the Grand minimum state. For example, data on $^{14}$C favor such an interpretation for the Dalton minimum. Unfortunately, sunspot data are quite incomplete for that period and not fully instructive: the quantity of the data is much worse than for the MM. Although a suppression of the cyclic activity is clearly visible, the interpretation of the event is not straightforward. In particular, a butterfly diagram cannot be constructed for the Dalton minimum, making it impossible to study the North–South asymmetry. The Dalton minimum was shorter than the MM but similar to it in its overall structure (sharp decrease followed by a gradual recovery). A tiny suppression of the activity (called sometimes the Modern minimum) around 1890 was even less pronounced and shorter than the Dalton minimum. The whole bulk of cosmogenic isotope data provides, however, additional support to the idea that the solar dynamo machine tries from time to time to go into the Grand minimum state, and that only a fraction of such attempts is successful.49 In addition to the above, an event occurred circa 1633 AD, for which observations by Gassendi imply asymmetric butterfly diagrams, provides another example of this kind.19

3.2. The heliosphere and cosmic rays

Because of the very low level of solar magnetic activity, the heliosphere (the region totally controlled by the solar wind and magnetic field) is expected to be quite different during the MM from its present state. It has been shown that while the modulation of cosmic rays was reduced during the MM, cosmic rays were still modulated,$^{15,50,51}$ which together with the fact that some magnetic activity was still observed during the MM, implies that the heliosphere did exist during that time, but probably its size was reduced.
Some estimates of the heliospheric parameters have been performed based on the available data sets discussed above. It is supposed\textsuperscript{52–54} that the solar wind was significantly slower during the MM, 200–350 km/s, compared to the presently measured 400–800 km/s. The interplanetary magnetic field (actually its $B_z$ component)\textsuperscript{54} and the axial dipole strength\textsuperscript{55} were also estimated to be essentially lower (by a factor 4–7) than presently. Applying a heliospheric model of cosmic ray transport to the measured $^{10}$Be in polar ice, Scherer \textit{et al.}\textsuperscript{56,57} have shown that the diffusion coefficient of cosmic rays in the heliosphere should be increased during the MM, which implies decreased level of the interplanetary magnetic field and/or interplanetary turbulence. However, these numbers were obtained using regression or other models based on sunspot numbers and fitted to modern conditions and, therefore, can be considered only as rough estimates.

It is not straightforward to interpret the observational facts discussed in the previous section in terms of the heliosphere. For instance, the dominating 22-year periodicity visible in $^{14}$C data during the MM can be understood in two ways. First, as suggested in Ref. 15, it may be due to a natural 22-year periodicity in the global solar magnetic parameters (solar wind, poloidal field, etc.). On the other hand, the modulation of cosmic rays can be drift-dominated contrary to the diffusion dominated modulation during normal solar activity times. Being only a relatively minor factor nowadays, polarity-dependent drifts\textsuperscript{58,59} may become dominant during the MM when diffusion/convection modulation is suppressed, so that the regular 22-year change of the global magnetic field polarity (assuming it was maintained throughout the MM) may alone lead to a basic 22-year cycle in low energy cosmic rays. A direct modeling of this effect forms a challenging task since the usual heliospheric models, which work well for the recent activity, cannot be applied to a Grand minimum. For example, a heliospheric model corresponding to the recent minima of solar cycle can not adequately reproduce the MM conditions because of some major distinctions. Within a solar cycle, there is a strong correlation between the level of sunspot activity and tilt of the heliospheric neutral sheet. However, this correlation does not correspond to a real physical link, and suppressed sunspot formation does not imply a flat neutral sheet during a Grand minimum. If the global magnetic field reversal keeps working during the MM, the tilt is expected to vary within its full extent.\textsuperscript{20} Also, a residual modulation beyond the termination shock may become important during a Grand minimum.\textsuperscript{57,60}
Resolving this problem is a challenging task, and a proper modeling of the heliospheric conditions during a Grand minimum is required to address this issue. Such work is in progress.

### 3.3. Other minima

Occurrence of a Grand minima is a rare but not unique phenomenon. The most recent Grand minima, other than the MM, are Spörer (around 1500 AD), Wolf (around 1300 AD) and the tiny Oort (around 1050 AD), but they appear more or less regular through millennia. However, only few of them can be studied in some detail, in addition to the MM.

The Spörer minimum (1415–1535 AD) was not covered by direct sunspot observations, but annual cosmogenic isotope records allow some conclusions about cyclic activity to be drawn. The data on $^{10}$Be in polar ice (Dye-3, Ref. 27) depict the dominant cyclicity with 20–25-year periodicity (see Fig. 7) with some power in the 5–10-year interval during the minimum. Radiocarbon $^{14}$C data imply periodicities at about 22 years (continuous), and 7–11 years (intermittent) during the Spörer minimum. It is important that the 11-year signal is not observed during 1460–1500, corresponding probably to the deep phase of the minimum. Therefore, both isotope series show the dominance of the 22-year cycle on the background of a greatly suppressed 11-year cycle during the Spörer minimum. This pattern is very similar to the MM, supporting the idea that Grand minima correspond to

![Fig. 7. The power spectrum (FFT) of the annual $^{10}$Be data from Dye-3 series.](image)
a specific state of the dynamo machine. No definite conclusion can be made on the exact cycle length though.

Another suppression of the solar activity is the Dalton minimum taken place during 1790s–1820s. Although we do not regard it as a Grand minimum (see discussion in Sec. 3.1), some of its features are quite similar to those of the MM. In particular, its start was also quite abrupt, especially if the lost cycle in 1790s is taken into account. The recovery to the normal level was gradual, through fairly regular 11-year cycles. Unfortunately, the quantity of the sunspot data available was very small in the beginning of the Dalton minimum, and no definite information on cycle lengths can be obtained. A general similarity between Maunder and Dalton minima may imply that the latter, while not a Grand minimum, corresponds to an attempt of the dynamo machine to switch into the minimum mode.

4. Conclusions
The main conclusion of our paper can be summarized as follows.

(1) The MM of solar activity is considered as a period of extraordinarily low sunspot occurrence. The activity did not, however, completely vanish. A vestige of the nominal 11-year cycle is visible in sunspots at the end of the MM, and traces of the 11- and 22-year cycles can be found in proxy data as approximately 14- and 28-year cycles during the whole MM. As far as we can distinguish activity tracers for the Northern and Southern hemispheres, the activity demonstrates a high level of North–South asymmetry during the MM. The Southern hemisphere appeared to be much more active than the Northern. While the transition into the deep minimum phase was abrupt — nearly instantaneous comparing to the typical cycle length — the recovery from the minimum was gradual and took several decades.

(2) The above phenomenological features of the MM give some hints about behavior of the solar dynamo machine during the MM times. On one hand, the start of the MM looks like a strong deviation/excursion of the solar dynamo action. On the other hand, it looks plausible that the cyclic dynamo kept working through the whole MM but producing rather unusual magnetic configurations with a strong North–South asymmetry.

(3) The MM and other long-term Grand minima should be distinguished from shorter events like, e.g., the Dalton minimum. We believe that
the solar dynamo machine tries to pass into a Grand minima state from time to time. Some of such attempts are successful and Grand minima occur while the unsuccessful attempts result in shorter events like Dalton minimum or even in minor events like a phase deviation that occurred circa 1900. In this context, in order to distinguish from such events, we would like to define a Grand minimum as a period with long-term (compared to the typical solar cycle length) and strong suppression of the solar magnetic activity.

Let us finish our presentation with some remarks concerning possible perspectives in the Grand minima study. An extensive comparative study of the Spörer and Maunder minima as well as the Dalton minimum would reveal features which are common and typical for a Grand minimum. According to the nature of the available data sources, the investigation is most promising in the field of analysis of cosmogenic isotope data. However, the most direct information comes from sunspot observations. We would like also note that even the most recent group sunspot number series, which replaces the famous but outdated Wolf sunspot series, is not complete, and some additional sunspot data are still to be restored from astronomical archive. In particular, we refer to the talk by Hoyt who stated that some sunspot observations by Scheiner, Alischer, Musano, Soemmering, Chevallier, and Williamson are still not included in the available databases. In particular, the Soemmering data known from look adequate to construct the butterfly diagram for the period 1826–1829, adjacent to the Dalton minimum. Note that a butterfly diagram, even if it is based on isolated data or obtained for a limited temporal interval, can still be extremely useful for the historical reconstruction of solar activity. For instance, the butterfly diagram built using Manfredi and Salvago’s observations at Bologna during 1703–1709 strongly supports the conclusion of a very strong North–South asymmetry of solar activity known from the La Hire data from Paris. Some archival data can be found also outside major observatories. For example, earlier unknown sunspot data from the Mexican astronomer J. A. Alzate and Portuguese astronomer Sanches Dorta have recently been discovered for the period 1784–1785, i.e., before the Dalton minimum when observations were quite scarce.

Concluding, the MM forms a challenging problem for both experimental and theoretical studies. The bulk of direct and indirect data about solar, heliospheric and terrestrial systems, together with theoretical developments lead to a better understanding of the phenomenon. During the last decades,
a substantial progress has been achieved in this direction but the puzzle is yet far from being solved.

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